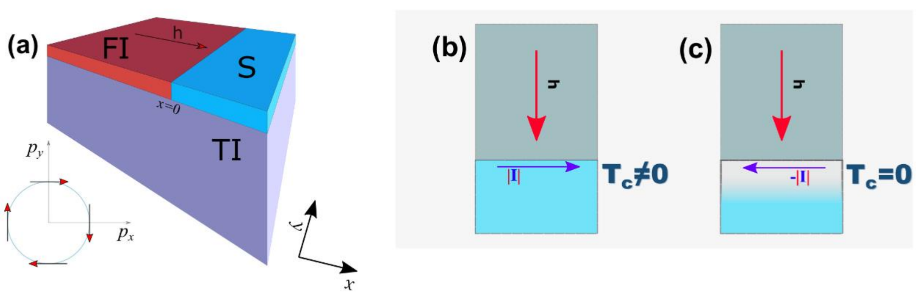
Hybrid helical state and superconducting diode effect in S/F/TI heterostructures

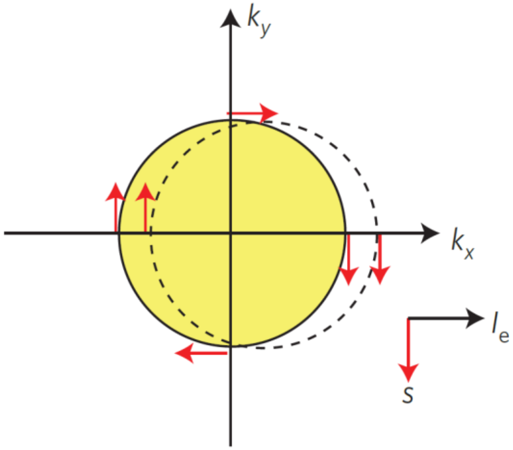
Andrey Vasenko

Moscow Institute of Electronics and Mathematics, HSE University, and Department of Theoretical Physics, Lebedev Physical Institute



Introduction: Helical state

- Breaking time-reversal symmetry (in-plane magnetic field: external or exchange)
- Breaking inversion symmetry (SOC)
- 2D superconductivity



$$\boldsymbol{n} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})$$

The helical state was originally predicted for two-dimensional systems with spin-orbit coupling (SOC) under the applied in-plane magnetic field. The SOC produces the spin-momentum locking term in the Hamiltonian. The applied field makes spin-down state energetically more favorable. Due to the spin-momentum locking it results in the fact that one of the mutually opposite momentum directions along the axis perpendicular to the Zeeman field is more favorable. That should lead to the appearance of the spontaneous current. However, the superconductor develops a phase gradient, which exactly compensates the spontaneous current. The resulting phase-inhomogeneous zero-current state is the true ground state of the system.

$$\Delta(\mathbf{r}) = \Delta e^{i\mathbf{q}_0\mathbf{r}}$$

Helical state in noncentrosymmetric S

PRL **94,** 137002 (2005) PHYSICAL REVIEW LETTERS week ending 8 APRIL 2005

Helical Vortex Phase in the Noncentrosymmetric $CePt_3Si$

R. P. Kaur, D. F. Agterberg, and M. Sigrist

Department of Physics, University of Wisconsin–Milwaukee, Milwaukee, Wisconsin 53211, USA
and Theoretische Physik ETH-Hönggerberg CH-8093 Zürich, Switzerland

(Received 4 August 2004; published 6 April 2005)

We consider the role of magnetic fields on the broken inversion superconductor $CePt_3Si$. We show that the upper critical field for a field along the c axis exhibits a much weaker paramagnetic effect than for a field applied perpendicular to the c axis. The in-plane paramagnetic effect is strongly reduced by the appearance of helical structure in the order parameter. We find that, to get good agreement between theory and recent experimental measurements of H_{c2}, this helical structure is required. We propose a Josephson junction experiment that can be used to detect this helical order. In particular, we predict that the Josephson current will exhibit a magnetic interference pattern for a magnetic field applied *perpendicular* to the junction normal. We also discuss unusual magnetic effects associated with the helical order.

DOI: 10.1103/PhysRevLett.94.137002 PACS numbers: 74.20.–z

$$\boldsymbol{q} = -2m\epsilon \boldsymbol{n} \times \boldsymbol{B}$$

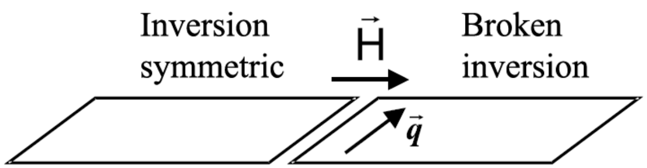


FIG. 2. Josephson junction geometry for the observation of a helical phase.

Helical state in the quasiclassical theory

PHYSICAL REVIEW B **92**, 014509 (2015)

Quasiclassical theory of disordered Rashba superconductors

Manuel Houzet and Julia S. Meyer
University of Grenoble Alpes, INAC-SPSMS, F-38000 Grenoble, France
and CEA, INAC-SPSMS, F-38000 Grenoble, France
(Received 16 February 2015; published 20 July 2015)

We derive the quasiclassical equations that describe two-dimensional superconductors with a large Rashba spin-orbit coupling and in the presence of impurities. These equations account for the helical phase induced by an in-plane magnetic field, with a superconducting order parameter that is spatially modulated along a direction perpendicular to the field. We also derive the generalized Ginzburg-Landau functional, which includes a linear-in-gradient term corresponding to the helical phase. This theory paves the way for studies of the proximity effect in two-dimensional electron gases with large spin-orbit coupling.

DOI: 10.1103/PhysRevB.92.014509 PACS number(s): 74.78.−w, 74.20.Mn, 74.25.Ha, 75.70.Tj

$$\mathbf{q} = -\frac{4\alpha}{\alpha^2 + v^2}(\boldsymbol{h}_{\parallel} \times \hat{\mathbf{z}})$$

Hybrid helical state and superconducting diode effect in S/F/TI heterostructures

T. Karabassov,[1,*] I. V. Bobkova,[1,2,3] A. A. Golubov,[4] and A. S. Vasenko[1,5]

[1]*HSE University, 101000 Moscow, Russia*
[2]*Institute of Solid State Physics, Chernogolovka, Moscow reg., 142432 Russia*
[3]*Moscow Institute of Physics and Technology, Dolgoprudny, 141700 Russia*
[4]*Faculty of Science and Technology and MESA+ Institute for Nanotechnology, University of Twente, 7500 AE Enschede, The Netherlands*
[5]*I.E. Tamm Department of Theoretical Physics, P.N. Lebedev Physical Institute, Russian Academy of Sciences, 119991 Moscow, Russia*

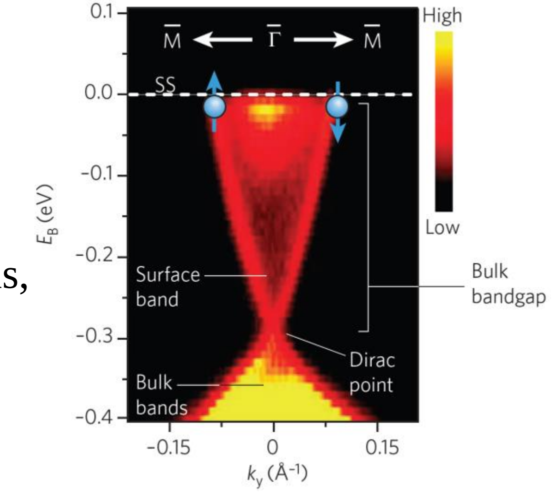
Topological insulator in a nutshell

…a new state of matter that has been predicted and discovered!

- Bulk is insulating; edge (2D)/ surface (3D) a very good conductor.
- Important ingredient: spin-orbit coupling:

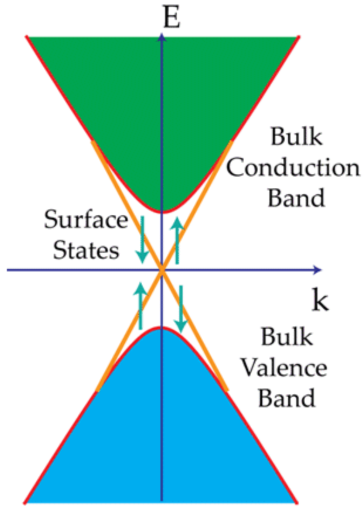
opposite force for opposite spins.

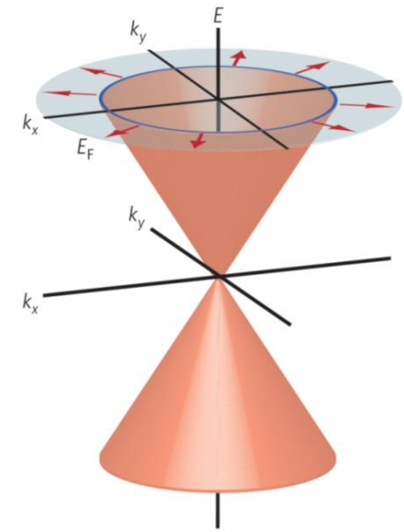
- Topological invariant is insensitive to any continuous deformation of Hamiltonian (topological protection): disorder, geometry, weak interactions, etc…



Examples:

- 2D: HgTe/CdTe; 3D: Bi_2Se_3, Bi_2Te_3 Sb_2Te_3, $TlBiSe_2$, Bi_2Te_2Se.





2D

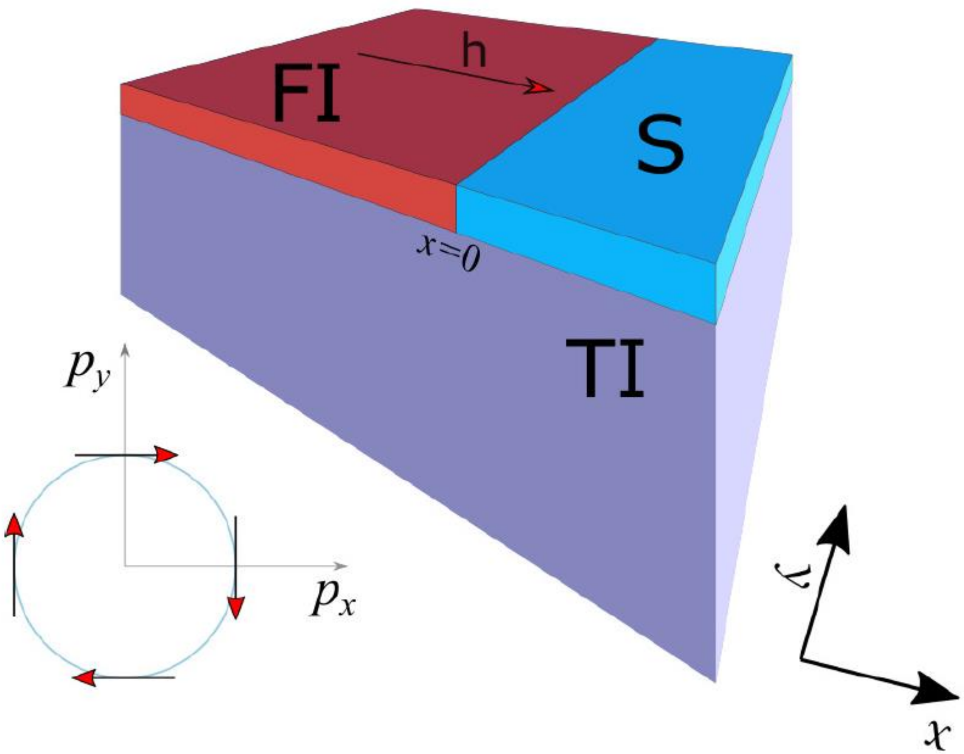
***Theo1**: C.L. Kane and E.J. Mele, **PRL** 95, 226801 (2005)*
***Theo2**: B.A. Bernevig et al., **Science** 314, 1757 (2006)*
***Exp**: M. König et al., **Science** 318, 766 (2007)*

***Theo**: L. Fu, et al., **PRL** 98, 106803 (2007)*
***Exp1**: Zhang H. et al., **Nat. Phys.** 5, 438 (2009)*
***Exp3**: S. Takafumi et al., **PRL** 105, 136802 (2010)*

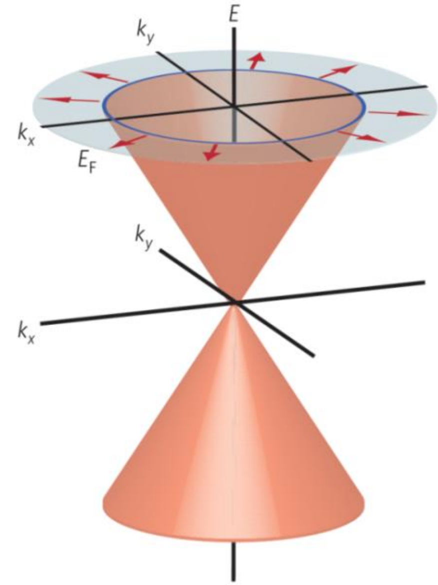
3D

S/F/TI hybrid structure: Hybrid helical state

It is found that although the exchange field and superconducting order parameter are spatially separated, the latter develops a spontaneous phase gradient, that is the finite-momentum helical state is realized. At the same time it is accompanied by the spontaneous currents, inhomogeneously distributed over the bilayer in such a way that the net current vanishes.



Spin-momentum locking

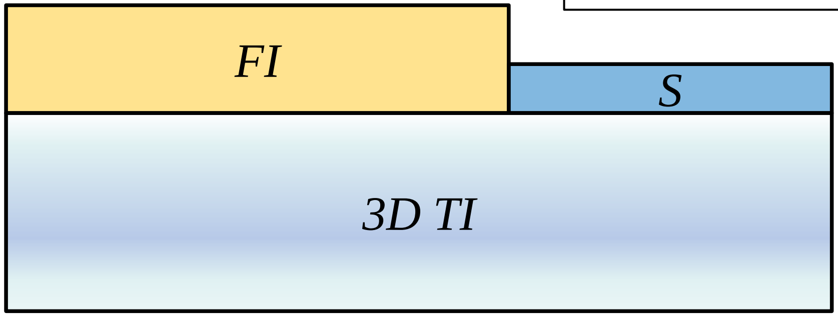


$$\boldsymbol{n} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})$$

Model

Linearized Usadel equations (dirty limit) in the limit of $T \leq T_c$

$$\xi_s^2 \pi T_{cs} \left(\partial_x^2 + \partial_y^2\right) f_s - |\omega_n| f_s + \Delta(\mathbf{r}) = 0$$



$$\Delta \ln \frac{T_{cs}}{T} = \pi T \sum_{\omega_n} \left(\frac{\Delta}{|\omega_n|} - f_s \right)$$

$$\left(\partial_x - \frac{2i}{\alpha} h_y\right)^2 f_T + \left(\partial_y + \frac{2i}{\alpha} h_x\right)^2 f_T = \frac{|\omega_n|}{\xi_n^2 \pi T_{cs}} f_T$$

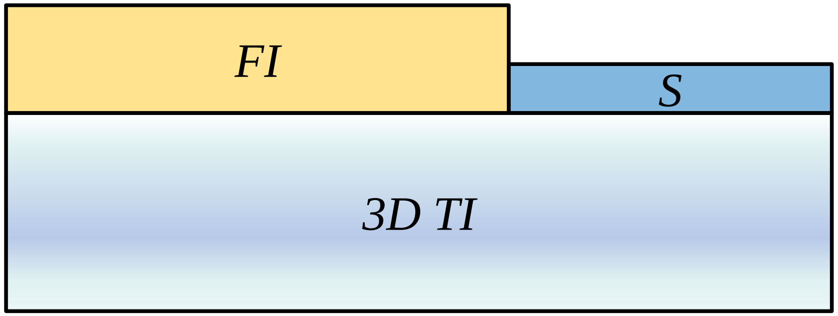
A. Zyuzin et al, Phys. Rev. B ***93****, 214502 (2016)*

h_y enters the solution as a phase factor $\exp(2ih_y x/\alpha)$.
Whereas h_x component has an impact on the critical temperature.

Ansatz for helical state and boundary conditions

The system is infinite in ***y*** *direction,*

$$\Delta(x, y) = \Delta(x)e^{iqy}$$



We use the KL boundary conditions,

$$\gamma_B \xi_n \frac{\partial f_T(0)}{\partial x} = f_s(0) - f_T(0),$$

$$\gamma \xi_n \frac{\partial f_T(0)}{\partial x} = \xi_s \frac{\partial f_s(0)}{\partial x}.$$

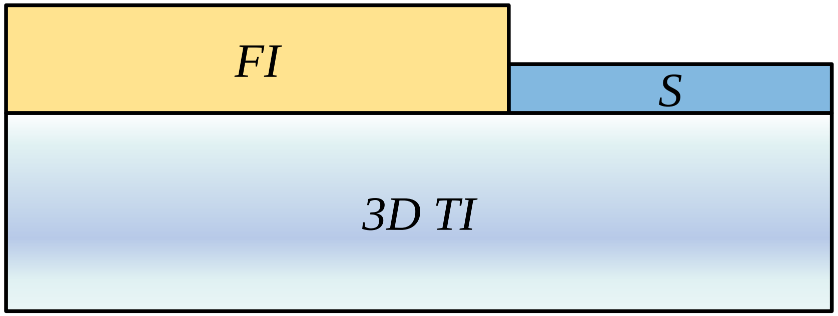
$\boxed{\gamma = \xi_s \sigma_n / \xi_n \sigma_s}$ *Proximity strength*

$\boxed{\gamma_B = R_b \sigma_n / \xi_n}$ *Interface barrier*

Supercurrent calculation

Nonlinear Usadel equation

$$D\hat{\nabla}\left(\hat{g}\hat{\nabla}\hat{g}\right) = \left[\omega_n \tau_z + i\hat{\Delta}, \hat{g}\right]$$



$$\hat{U} = \exp\left(iqy\tau_z/2\right)$$

$$\hat{g} = \hat{U}\hat{g}_q\hat{U}^\dagger$$

$$\hat{g}_q = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

Self-consistency equation

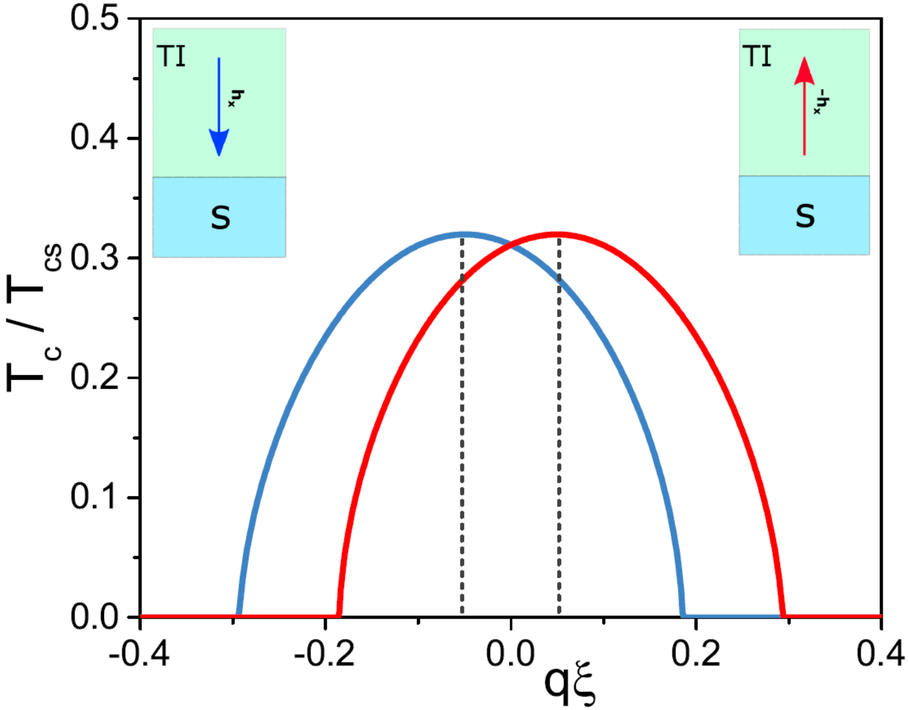
$$\Delta(x)\ln\frac{T_{cs}}{T} = \pi T \sum_{\omega_n>0}\left(\frac{\Delta(x)}{\omega_n} - 2\sin\theta_s\right)$$

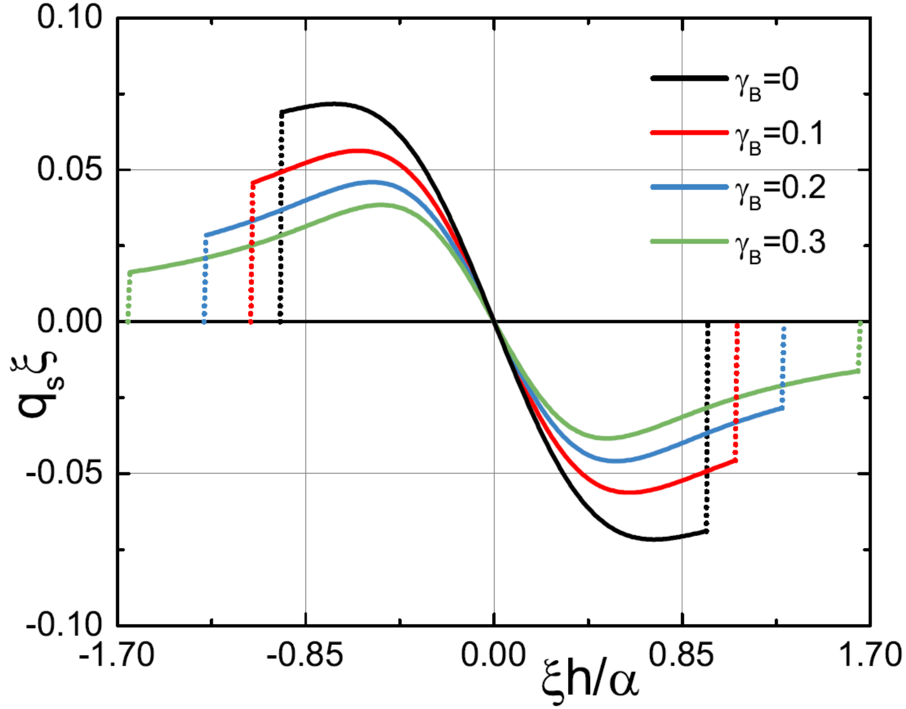
Hybrid helical state

Ground state of the system: non-zero q

$$q_s = q\left(\max\left[T_c(q)\right]\right)$$

$$\gamma_B = R_b \sigma_n / \xi_n$$





The supercurrent caused by q_s exactly compensates the supercurrent flowing on the TI surface in the opposite direction. For the transparent interface the pair momentum q_s is the most pronounced. It reflects the necessity of the proximity to the FI layer to produce the hybrid helical state. Abrupt drop to zero of the parameter q_s reflects the transition from superconducting to normal state.

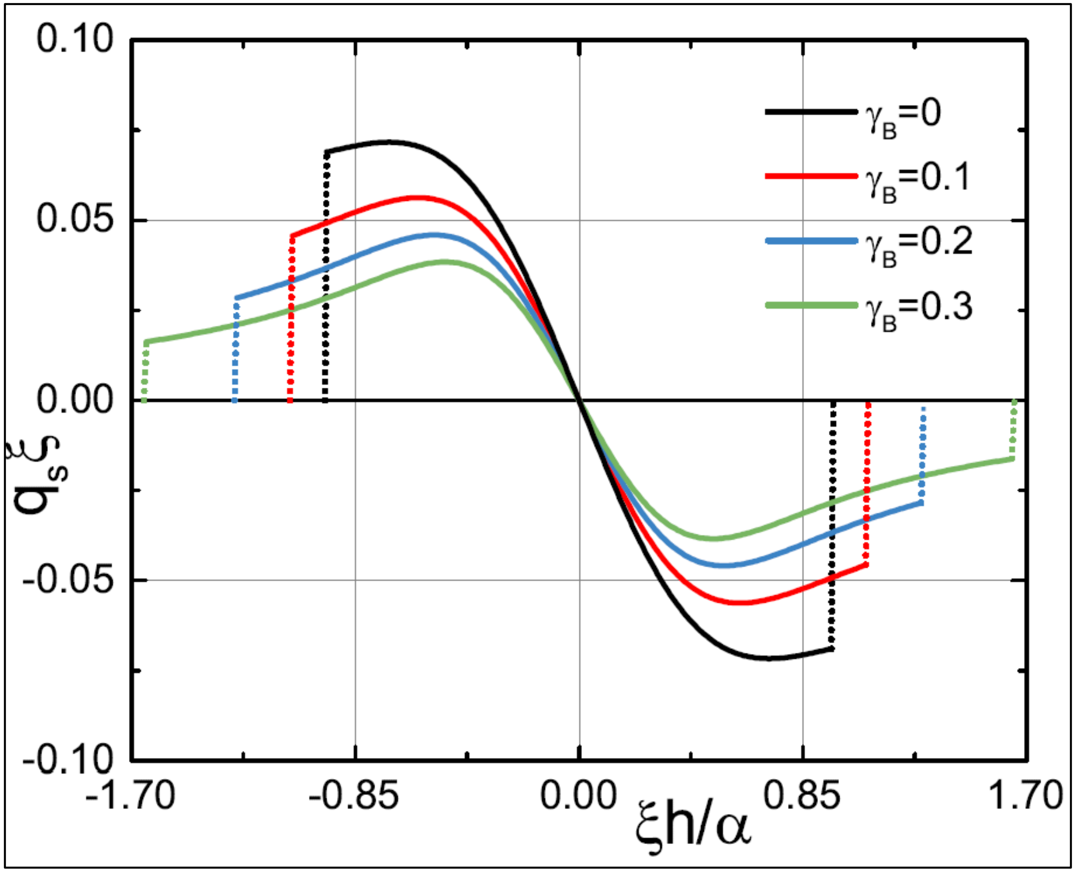
Case of $\Delta = const, d_s, d_f \ll \xi$ and $\gamma_B = 0$

$$q_s = -\frac{\gamma H d_f}{d_s + \gamma d_f}$$

$$H = 2\xi h/\alpha$$

$$q_s \propto -\gamma H d_f/d_s$$

q_s *is an odd function of* h



Ground state of the system: total supercurrent

$$I = \left[\int_{-d_f}^{0} \mathbf{j}^T(x)dx + \int_{0}^{d_s} \mathbf{j}^S(x)dx \right]$$

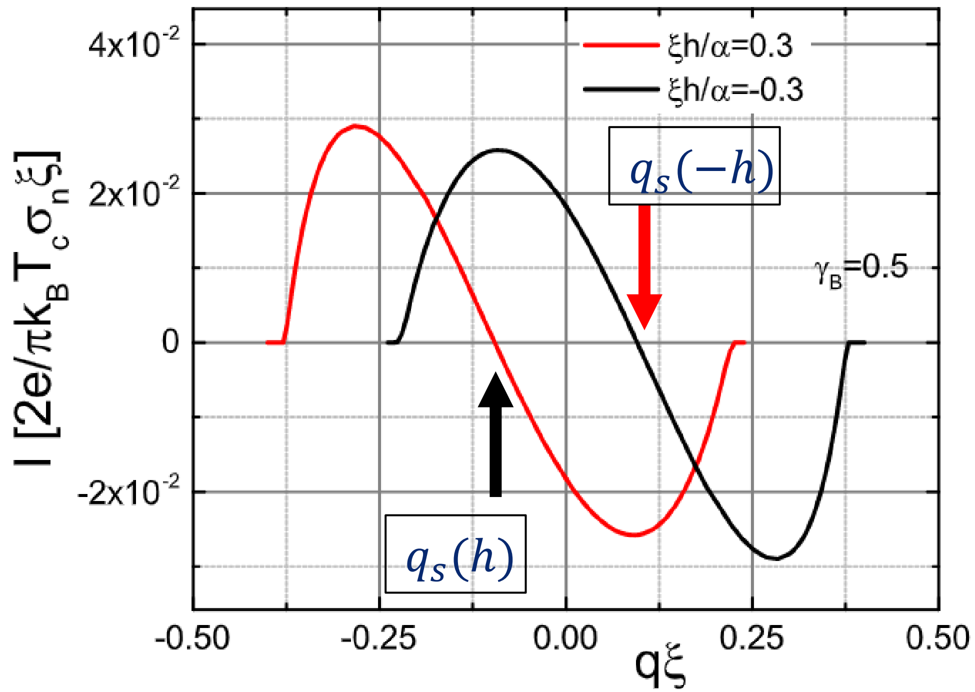
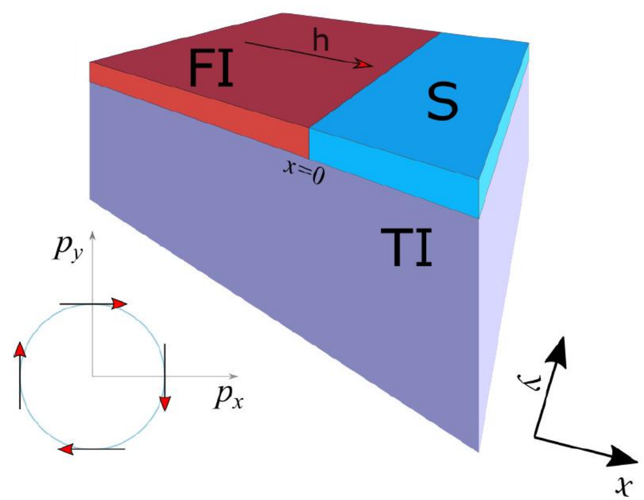


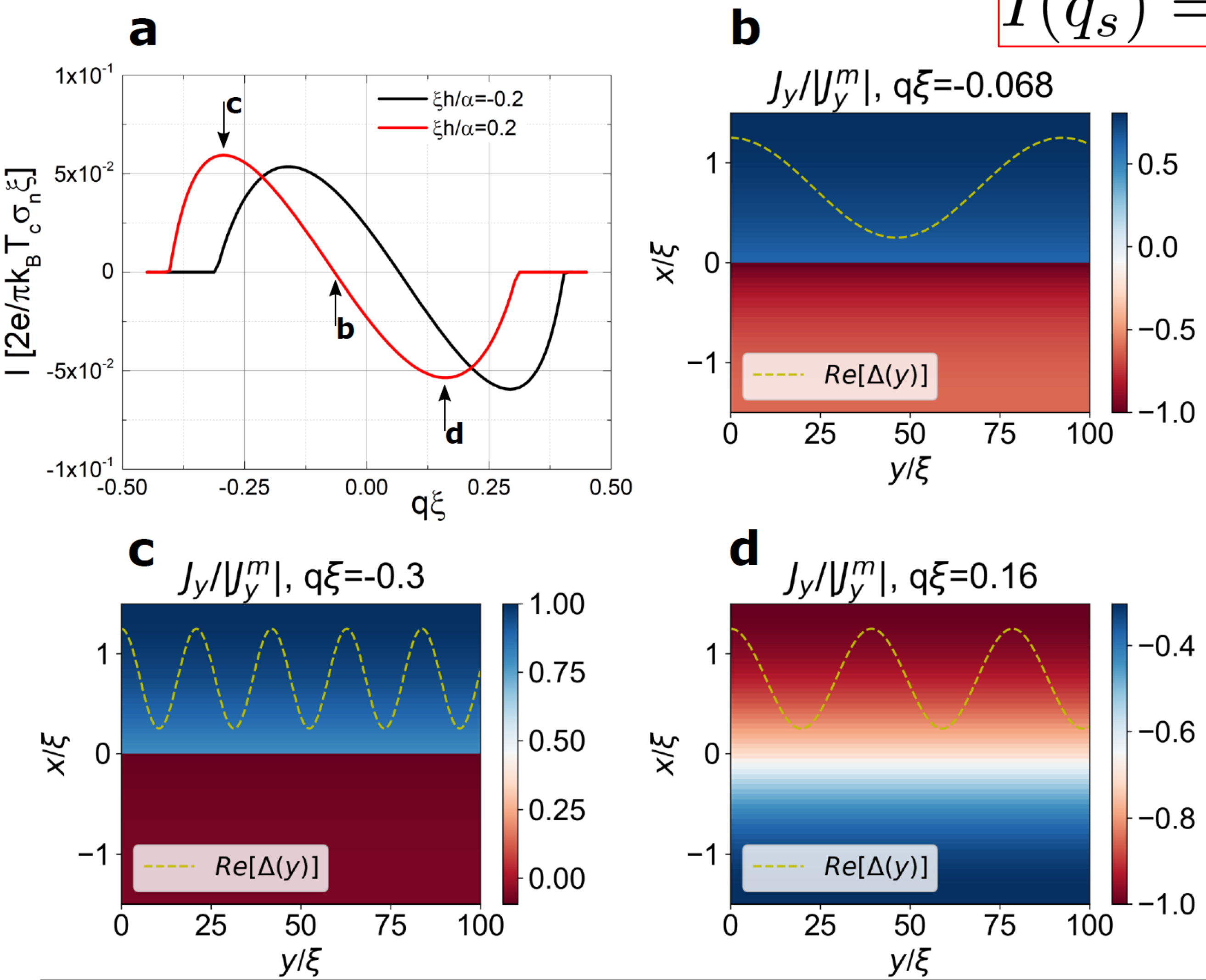
FIG. 4. The normalized supercurrent as a function of q calculated at temperature $T = 0.1T_{cs}$. The parameters of the S/TI interface: $\gamma = 0.5$, $\gamma_B = 0.5$.



$$I(q_s) = 0$$

Current distribution

$$I(q_s) = 0$$

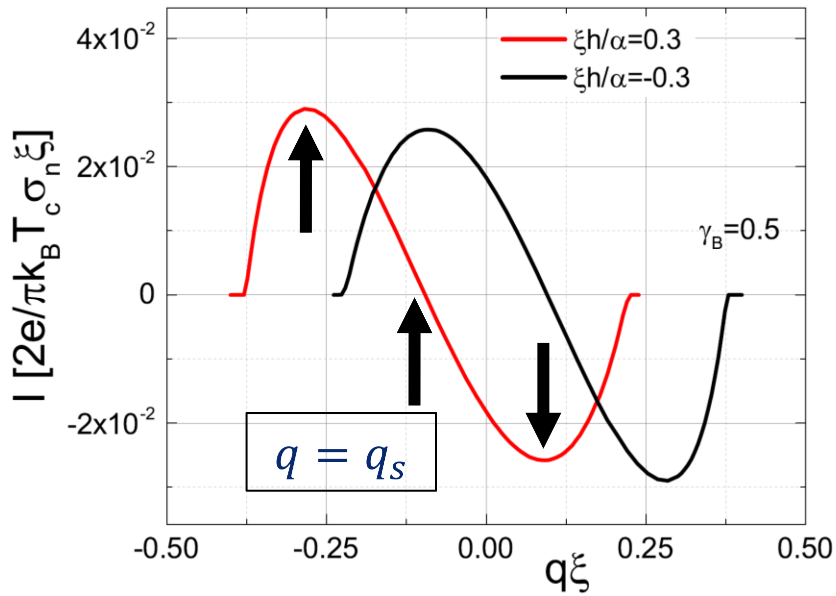


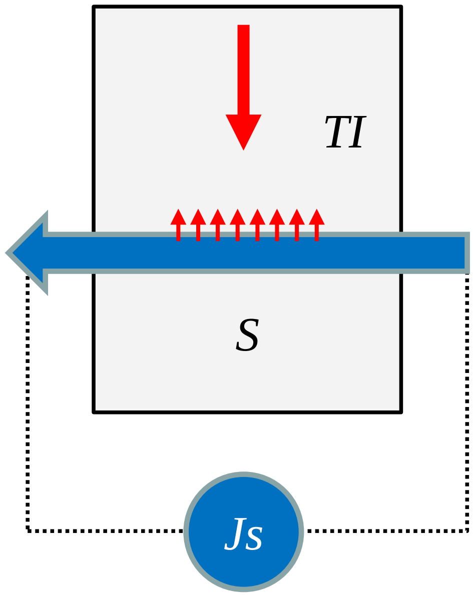
-Zero total current I

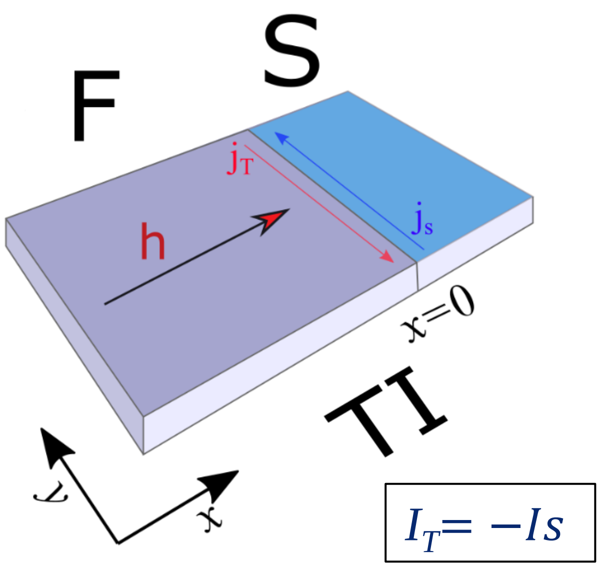
-Nonuniform current distribution $J_y(x, y)$

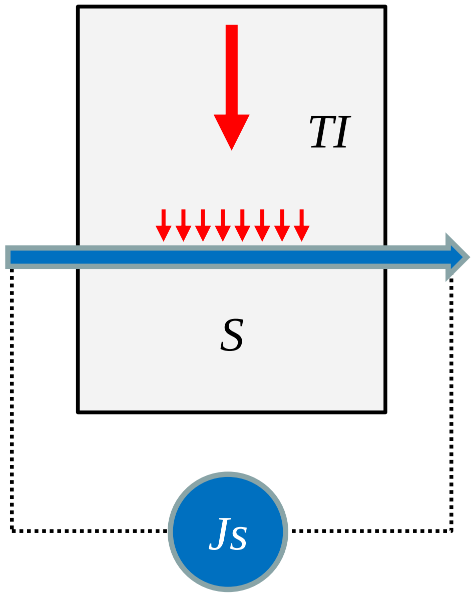
Critical current nonreciprocity and Superconducting Diode Effect

Current nonreciprocity









The superconducting diode effect

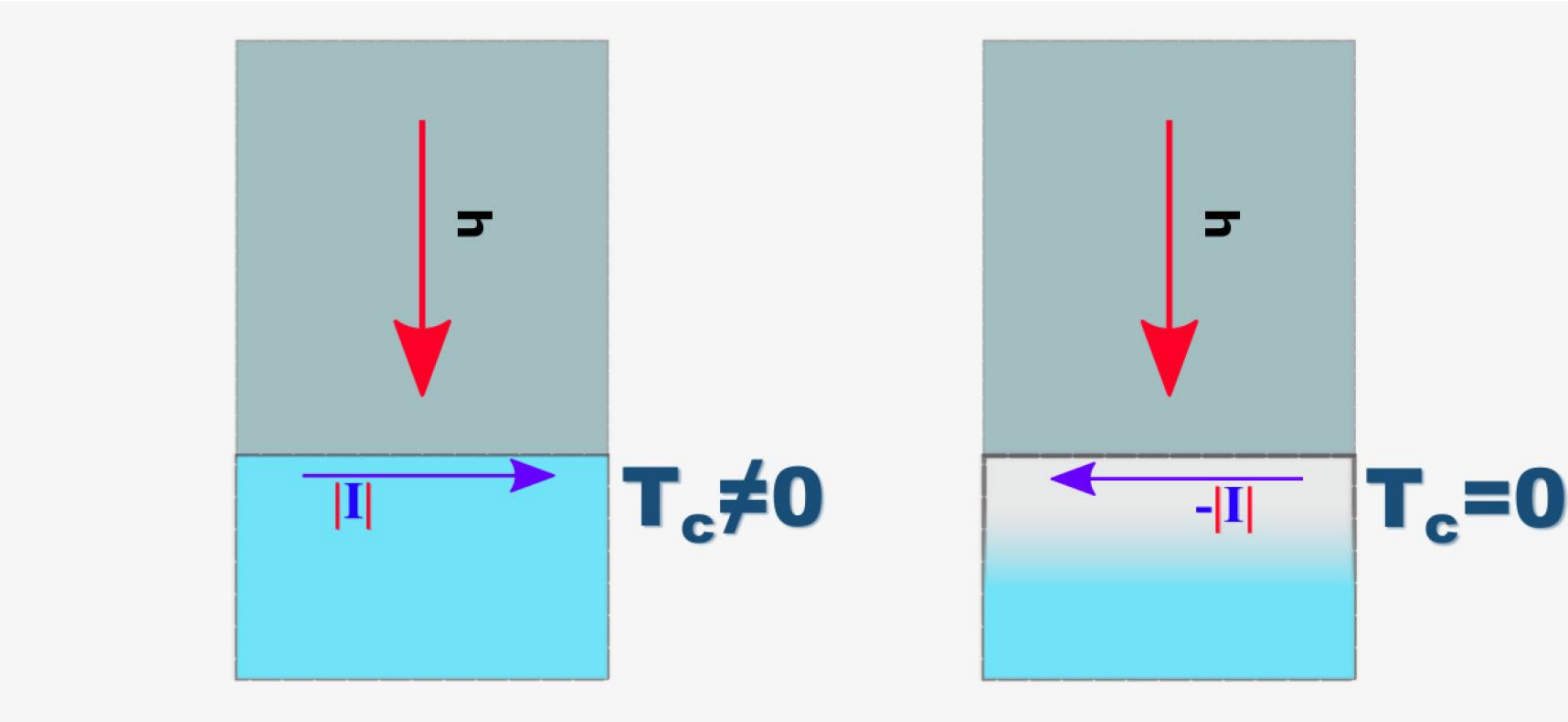
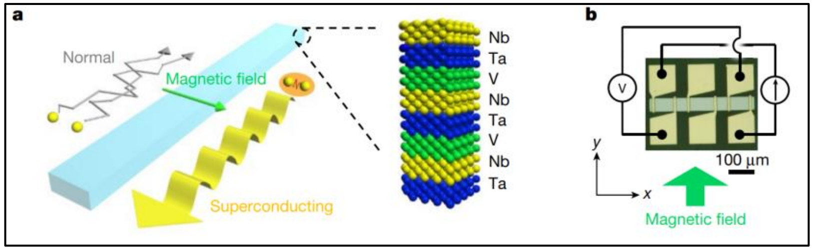


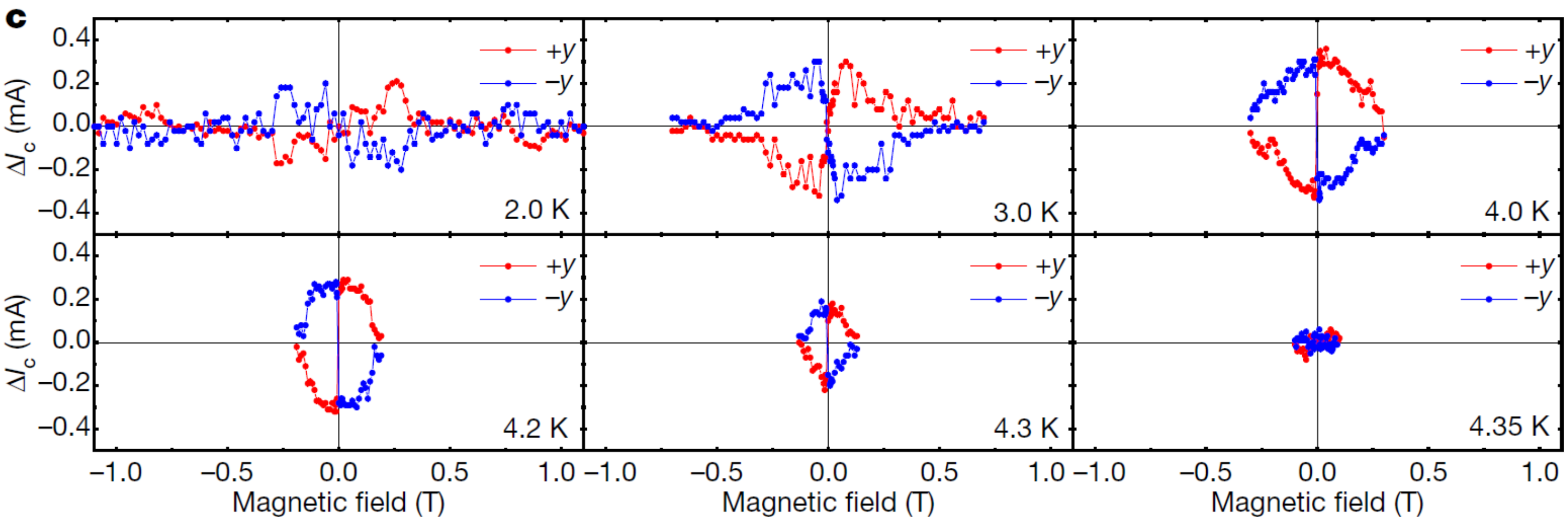
Illustration of the superconducting diode effect. Applying external supercurrent along the interface in one direction keeps the non-zero critical temperature (left), while reversing the current may completely destroy the superconducting state (right).

Superconducting diode effect: observation

- In superconducting layered systems



Ando, F., et al. Observation of superconducting diode effect. Nature ***584,*** *373–376 (2020)*



c, The nonreciprocal component of the critical current ΔI_c plotted as a function of the magnetic field at various temperatures. As the temperature increases towards the T_c, the ΔI_c clearly appears and subsequently shrinks.

Intrinsic Superconducting Diode Effect: theory

- In materials with broken TRS and inversion symmetry

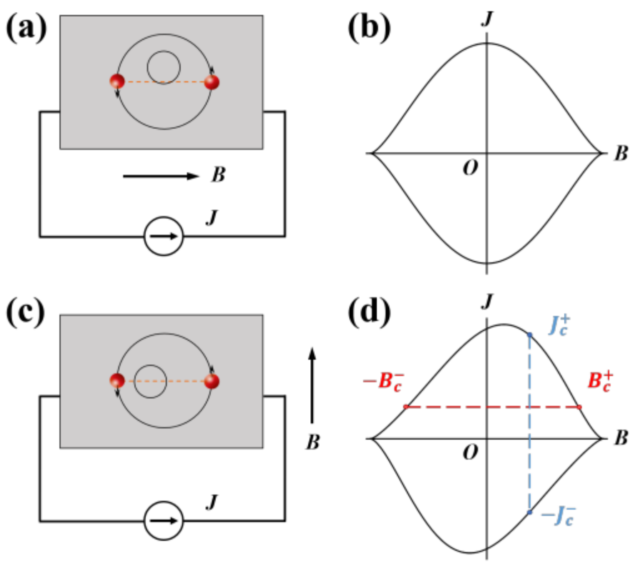
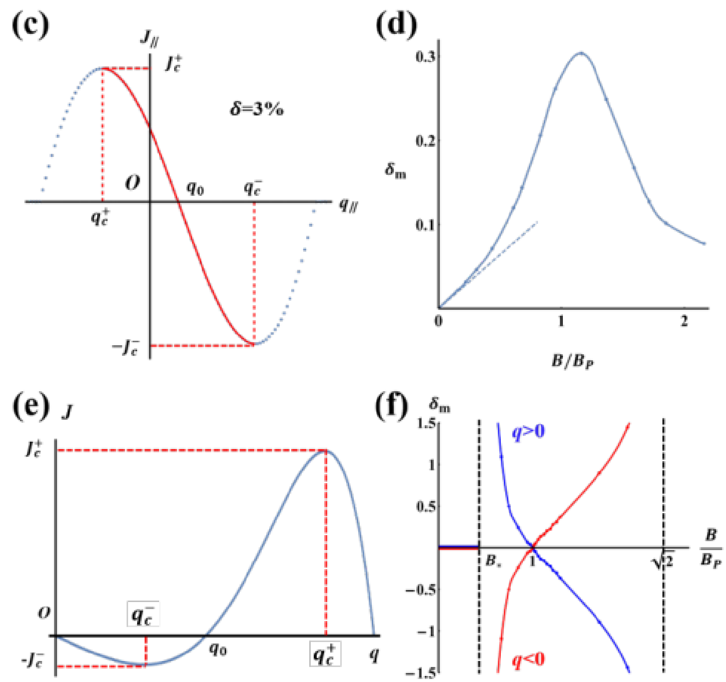


FIG. 1: Supercurrent diode effect in a Rashba superconductor under in-plane magnetic field B and external current source J. Panels (a,c) are device plots with circles denoting normal state Fermi surfaces, and (b,d) denote schematic phase diagram in B-J plane. When $\boldsymbol{B} \parallel \boldsymbol{J}$ in (a), the phase diagram in (b) is symmetric with respect to both B and J axes. And when $\boldsymbol{B} \perp \boldsymbol{J}$ in (c), the phase diagram in (d) is skewed, indicating nonreciprocal critical current $J_c^+ \neq J_c^-$ and polarity-dependent critical field $B_c^+ \neq B_c^-$.



2D helical superconductors

N. F. Q. Yuan and L. Fu, Supercurrent diode effect, PNAS, ***119****, 15 (2022)*

Intrinsic Superconducting Diode Effect: theory

- In materials with broken TRS and inversion symmetry

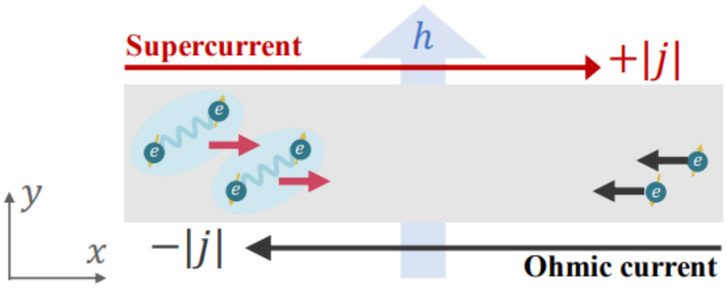


FIG. 1. Schematic figure for the SDE. The system has zero and finite resistance for the rightward and leftward current, respectively, and *vice versa* when the magnetic field h is reversed.

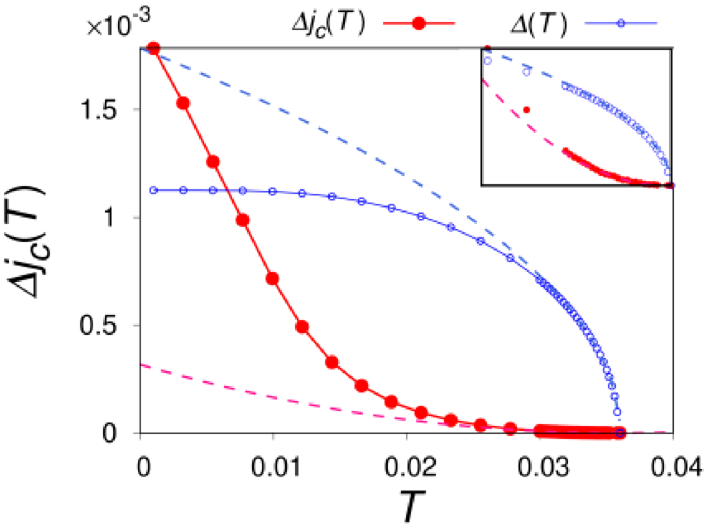
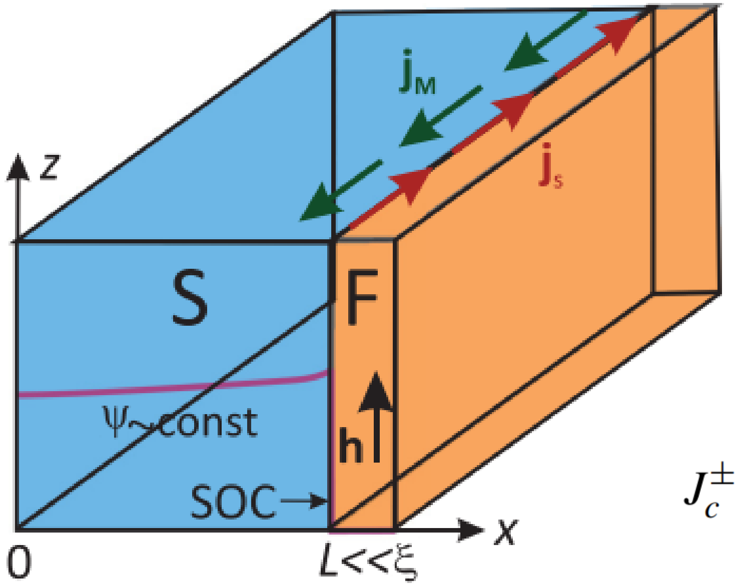


FIG. 2. The temperature dependence of Δj_c at $h = 0.03$. The red closed circles indicate $\Delta j_c(T)$, while the open blue circles indicate $\Delta(T)$ (a.u.). The dashed lines show the fitting curve of $\Delta j_c(T)$ and $\Delta(T)$ near T_c with $(T_c - T)^2$ and $\sqrt{T_c - T}$, respectively. The inset shows the enlarged figure near $T_c \simeq 0.036$.

A. Daido et al. Phys. Rev. Lett. ***128****, 037001 (2022)*

Superconducting Diode Effect: theory

- In S/F bilayers with SOC



$$J_c^{\pm} = \frac{\sqrt{2}LcH_{cm}\tau^{3/2}}{6\pi\sqrt{3}\lambda_0}\left(1 \pm \Delta H \frac{L\sqrt{\tau}}{2\sqrt{6}H_{cm}\lambda_0}\right)$$

FIG. 1. Sketch of a superconducting film placed in contact with a thin ferromagnetic layer. The spin-orbit coupling at the S/F interface produces a spontaneous supercurrent j_s, causing the increase of the superconducting order parameter inside the superconductor.

Zh. Devizorova et al. Phys. Rev. B ***103****, 064504 (2021)*

Superconducting diode effect in S/FI/TI

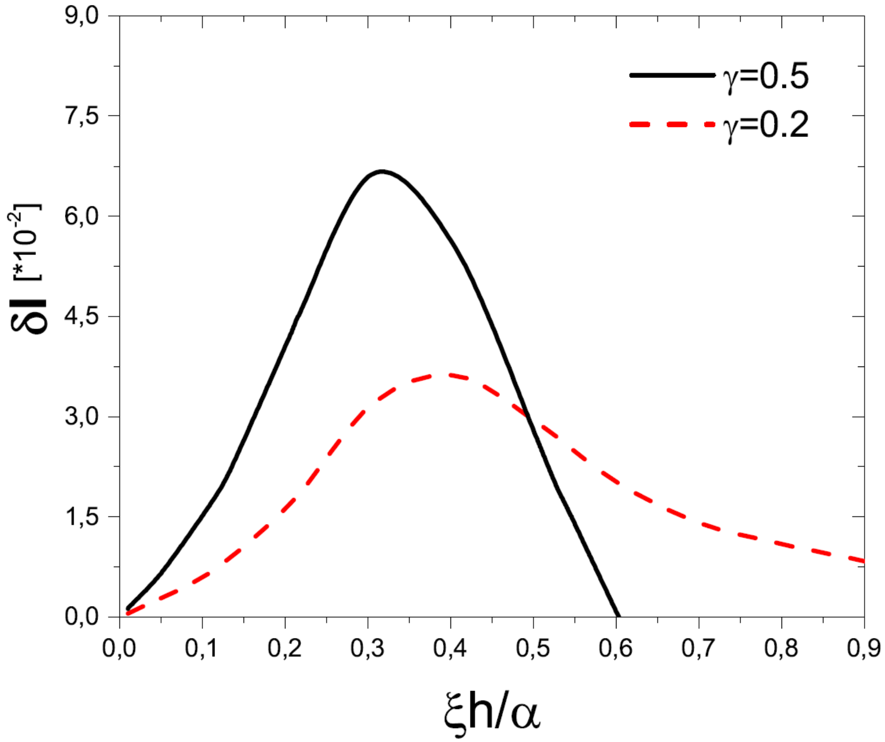


FIG. 6. δI as a function of magnetization h calculated at two different γ at $T = 0.1T_{cs}$. The interface parameter $\gamma_B = 0.5$

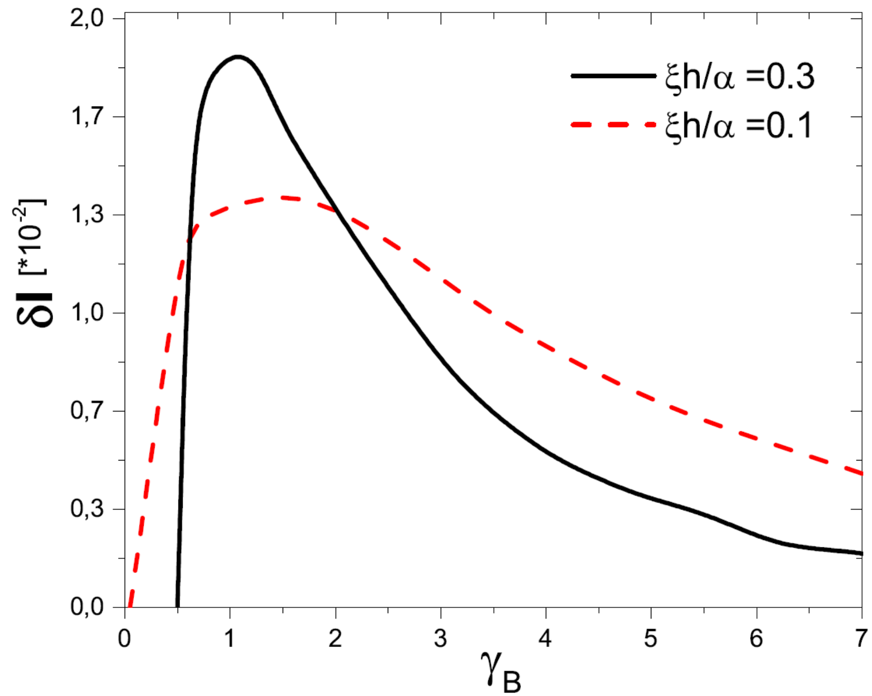


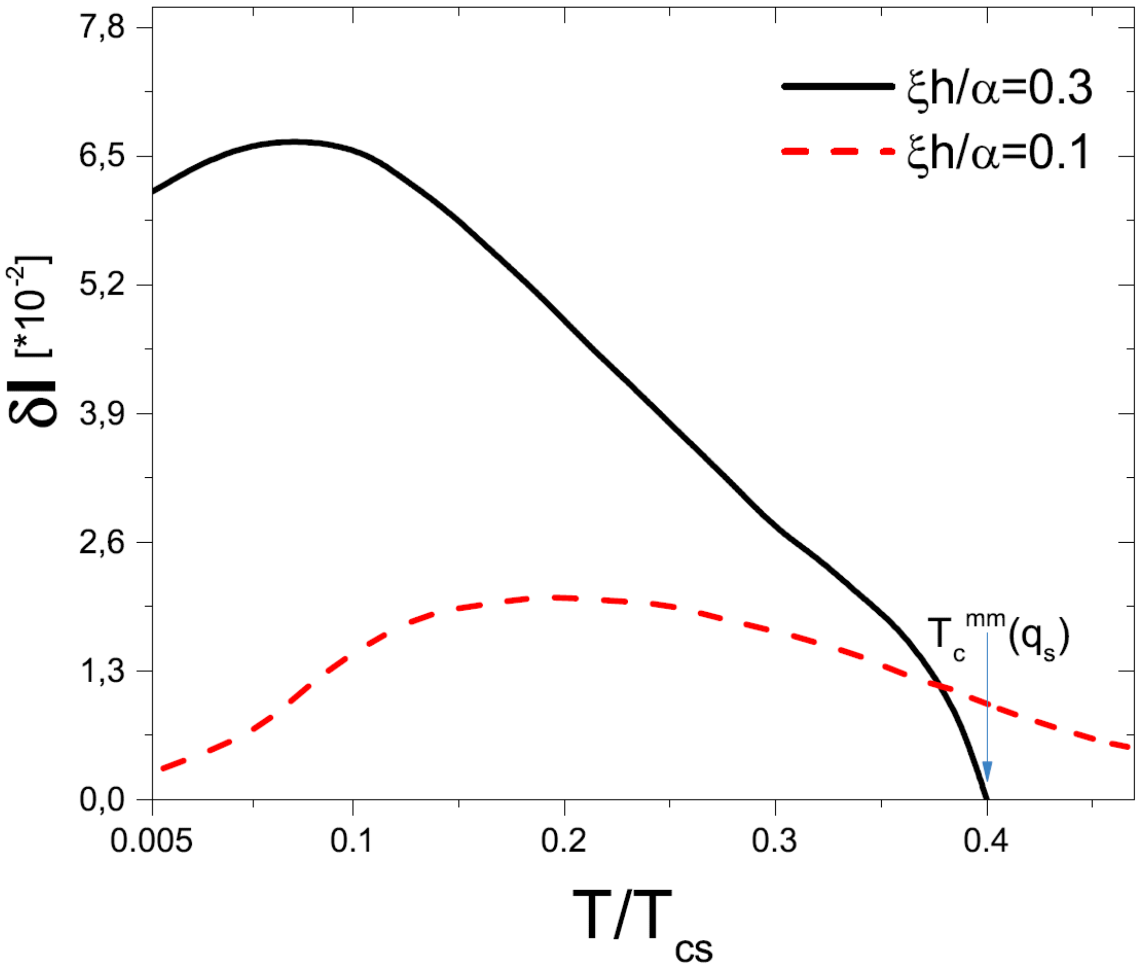
FIG. 8. δI as a function of transparency parameter γ_B calculated at two different h. The temperature is taken as $T = 0.4T_{cs}$ and $\gamma = 0.5$

$$\delta I = \frac{I_{c+} - I_{c-}}{I_{c+} + I_{c-}}$$

$$\gamma = \xi_s \sigma_n / \xi_n \sigma_s$$

$$\gamma_B = R_b \sigma_n / \xi_n$$

Superconducting diode effect in S/FI/TI

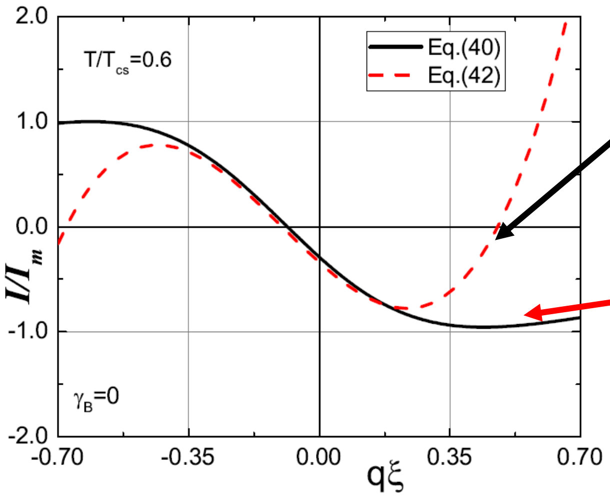


$$\delta I = \frac{I_{c+} - I_{c-}}{I_{c+} + I_{c-}}$$

FIG. 7. δI as a function of temperature T calculated at two different γ. Here T_c^{mm} is the transition temperature obtained via the multimode approach.

Analytical result

- thin S layer $\Delta = const \quad q\xi \ll 1$



$$I = -\frac{\pi \Delta_{eff}^2 \sigma T}{2e}\left(a_0 + a_1 q\xi + a_2 (q\xi)^2 + a_3 (q\xi)^3\right)$$

$$I = \frac{\pi \Delta^2 \sigma_s T}{2e} \sum \frac{I_T\left(q + 2h/\alpha\right) + I_s q}{(\omega_n/\xi_s^2 \pi T_{cs} + q^2)^2}$$

$$I_s = d_s - 2P\frac{A_{qs}}{k_{qs}^2} + P^2\left(\frac{d_s}{2\cosh^2 k_{qs} d_s} + \frac{A_{qs}}{2k_{qs}^2}\right)$$

$$I_T = \gamma\left(\frac{1-P}{\gamma_b + A_{qT}}\right)^2\left(\frac{d_f}{2k_q^2 \xi_n^2 \sinh^2 k_q d_f} + \frac{\coth k_q d_f}{2k_q^3 \xi_n^2}\right)$$

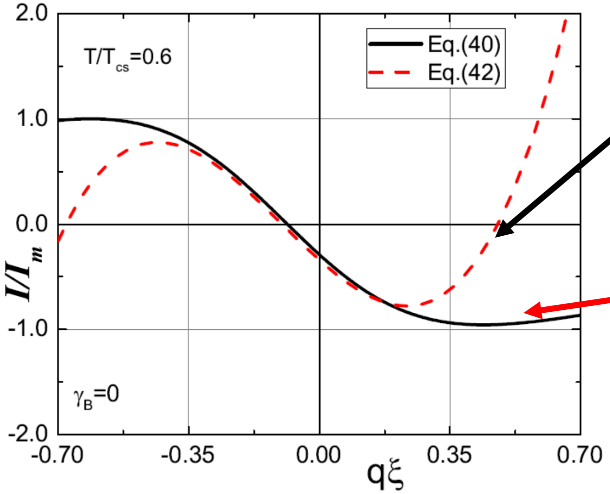
$$H d_f/\xi \ll 1$$

$$\delta I \approx \frac{1}{2}\frac{\sqrt{7\zeta(2)\zeta(3)}}{(T/T_{cs})^{5/2}}\frac{H d_f}{d_s} \approx 1.86 \frac{1}{(T/T_{cs})^{5/2}}\frac{H d_f}{d_s}$$

$$H = 2\xi h/\alpha$$

Analytical result

- thin S layer $\Delta = const \quad q\xi \ll 1$



$$I = -\frac{\pi\Delta_{eff}^2\sigma T}{2e}\left(a_0 + a_1 q\xi + a_2(q\xi)^2 + a_3(q\xi)^3\right)$$

$$I = \frac{\pi\Delta^2\sigma_s T}{2e}\sum\frac{I_T\left(q + 2h/\alpha\right) + I_s q}{(\omega_n/\xi_s^2\pi T_{cs} + q^2)^2}$$

$$I_s = d_s - 2P\frac{A_{qs}}{k_{qs}^2} + P^2\left(\frac{d_s}{2\cosh^2 k_{qs}d_s} + \frac{A_{qs}}{2k_{qs}^2}\right)$$

$$I_T = \gamma\left(\frac{1-P}{\gamma_b + A_{qT}}\right)^2\left(\frac{d_f}{2k_q^2\xi_n^2\sinh^2 k_q d_f} + \frac{\coth k_q d_f}{2k_q^3\xi_n^2}\right)$$

$$Hd_f/\xi \ll 1$$

$$\delta I \approx \frac{1}{2}\frac{\sqrt{7\zeta(2)\zeta(3)}}{(T/T_{cs})^{5/2}}\frac{Hd_f}{d_s} \approx 1.86\frac{1}{(T/T_{cs})^{5/2}}\frac{Hd_f}{d_s}$$

$$q_s \propto -\gamma Hd_f/d_s$$

$$H = 2\xi h/\alpha$$

SDE is controlled by the product Hd_f/d_s

Review

- The ground state of the system is characterized by Δ modulated with finite momentum q_s;
- This state is accompanied by the non-zero current distribution and zero average value;
- The hybrid helical state is responsible for nonreciprocity of I_c which manifests itself in the SDE;
- Some important analytical results have been derived, revealing controlling parameters and temperature dependence of the SDE.

Thank you!